# Neutrino oscillations in the early universe. Resonant case.


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**Abstract**

Lepton asymmetry generated in the early universe by neutrino oscillations into sterile partners is calculated. Kinetic equations are analytically reduced to a simple form that permits an easy numerical treatment. Asymptotic values of the asymmetry are at the level of 0.2-0.3 and are reasonably close to those obtained by other groups, though the approach to asymptotics in some cases is noticeably slower. No chaoticity is observed.


# 1 Introduction.

Neutrino oscillations in the early universe might possess a very interesting property, if active neutrinos ($\nu_a = \nu_e, \nu_\mu, \nu_\tau$) are mixed with sterile ones ($\nu_s$). Refraction index of neutrinos in the cosmic plasma depends upon the cosmological charge asymmetry of the plasma, $\eta$, which is normally quite small, $\eta = 10^{-9} - 10^{-10}$. Because of this dependence the transformation of $\nu_a$ into $\nu_s$ might be slightly more favorable than the transformation of the corresponding antineutrinos (or vice versa depending upon the sign of the initial asymmetry). The feedback effect is positive and leads to a further increase of asymmetry making, say, $\nu_a \rightarrow \nu_s$ transformation more and more efficient in comparison with $\bar{\nu}_a \rightarrow \bar{\nu}_s$. Of course the total leptonic charge of active plus sterile neutrinos is conserved but refraction index depends only on charge asymmetry in the sector of active neutrinos, and the lepton asymmetry in active sector could strongly increase.


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The effect of asymmetry generation takes place only for a sufficiently small mixing; i.e. for $\delta m^2 \sim 1$ eV$^2$ it is roughly bounded by $(\sin 2\theta)^2 \leq (\sim 10^{-3})$, where $\theta$ is the vacuum mixing angle. For large mixings active and sterile states of both neutrinos and antineutrinos would quickly reach thermally equilibrium values, they would become equally populated, and this prevents from generation of a large asymmetry. For a positive mass difference between $\nu_s$ and $\nu_a$ ($\delta m^2 > 0$) the asymmetry would remain small also in the case of small mixing. However for $\delta m^2 < 0$ the resonance MSW-transition [1, 2] might take place in the primeval plasma and this effect could compensate a smallness of vacuum mixing and produce a considerable lepton asymmetry in the sector of active neutrinos.

The instability with respect to generation of lepton asymmetry by neutrino oscillations was noticed in ref. [3] but it was concluded there, on the basis of simplified arguments, that the rise of asymmetry was terminated when it was still quite small. This conclusion was reconsidered in ref. [4] (see also refs. [5]-[8]) where it was argued that a very large asymmetry, close to 1, may be generated by the oscillations. This result was questioned in ref. [9] where it was claimed that back reaction effects are very strong, they significantly slowed down the rise of asymmetry, and the latter could reach only the value around $10^{-6} - 10^{-4}$ for reasonably small mass differences. However certain drawbacks of the approach of the paper [9] were indicated in ref. [10] (see also [11, 12]) and though not all of them were relevant, it would be enough to have one weak point to destroy a conclusion.

Since the result of a large asymmetry generation is very interesting and important (in particular, for the big bang nucleosynthesis, BBN) it is worthwhile to make an independent calculation of the effect. In this work kinetic equations governing neutrino oscillations in the early universe are analytically transformed to a rather simple form which after well controlled approximate simplifications permits an easy numerical solution.

The results of this work are reasonably close to those found in refs. [5, 6, 8, 10] for the case of $(\nu_{\mu,\tau} - \nu_s)$-oscillations and approximately 2-3 times smaller than the asymmetry found in ref. [13] for $(\nu_e - \nu_s)$-oscillations in the temperature range essential for BBN.

Another interesting issue related to the asymmetry generation is its possible chaoticity so that the sign of the final large value of the lepton asymmetry is a rapidly oscillating function of the oscillation parameters or of a small variation of the initial value of the cosmological charge asymmetry [14]-[21]. According to the calculations of the present paper, no chaoticity exists at least in the parameter range considered below, for which a large lepton asymmetry is generated.

## 2 Kinetic equations.

We assume that a non-negligible mixing exists only between one active and one sterile neutrino, so that the neutrino state is described by $2 \times 2$-density matrix $\rho$. Its evolution is governed by the usual equation:

$$i\dot{\rho} = [\mathcal{H}, \rho] \tag{1}$$

where $\mathcal{H}$ is the neutrino Hamiltonian. It contains the free part: which is diagonal in the mass basis:

$$\mathcal{H}_{free} = \mathrm{diag}\left[\sqrt{p^2 + m_a^2},\ \sqrt{p^2 + m_s^2}\right] \tag{2}$$

and the part that describes interaction with medium which looks simpler in the flavor basis. The interaction Hamiltonian contains first order terms given by refraction index (or effective potential of neutrinos). It was originally calculated in ref. [22] and consists of two terms (see also discussion in papers [23, 3]:

$$V_{eff}^a = \pm C_1 \eta^{(a)} G_F T^3 + C_2^a \frac{G_F^2 T^4 E}{\alpha} \ , \tag{3}$$

where $E$ is the neutrino energy, $T$ is the temperature of the plasma, $G_F = 1.166 \cdot 10^{-5}$ GeV$^{-2}$ is the Fermi coupling constant, $\alpha = 1/137$ is the fine structure constant, and the signs "±" refer to anti-neutrinos and neutrinos respectively. According to ref. [22] the coefficients $C_j$ are: $C_1 \approx 0.95$, $C_2^e \approx 0.61$ and $C_2^{\mu,\tau} \approx 0.17$. These values are true in the case of thermal equilibrium, otherwise these coefficients are some integrals over the distribution functions. The contributions to $\eta^{(a)}$ from different particle species are the following:

$$\eta^{(e)} = 2\eta_{\nu_e} + \eta_{\nu_\mu} + \eta_{\nu_\tau} + \eta_e - \eta_n/2 \ \ (\text{for } \nu_e) \ , \tag{4}$$

$$\eta^{(\mu)} = 2\eta_{\nu_\mu} + \eta_{\nu_e} + \eta_{\nu_\tau} - \eta_n/2 \ \ (\text{for } \nu_\mu) \ , \tag{5}$$

and $\eta^{(\tau)}$ for $\nu_\tau$ is obtained from eq. (5) by the interchange $\mu \leftrightarrow \tau$. The individual charge asymmetries, $\eta_X$, are defined as the ratio of the difference between particle-antiparticle number densities to the number density of photons:

$$\eta_X = \left(N_X - N_{\bar{X}}\right)/N_\gamma \tag{6}$$

The interaction Hamiltonian contains also the so called second order terms that describe the loss of coherence due to elastic or inelastic neutrino scattering and annihilation as well as neutrino production by reactions in the primeval plasma. For the description of the loss of coherence it is enough to take into account only imaginary part of the second order Hamiltonian. Discussion and calculations can be found in the papers [24]-[32]. The exact form of the second order terms has quite complicated matrix structure, it is non-linear in $\rho$, and contains multi-dimensional integrals over phase space (which can be reduced down to two dimensions). Their explicit expressions can be found e.g. in refs. [25, 29]. However it is very difficult, to solve kinetic equations for density matrix with the exact expressions for the second order terms, especially in the resonance case. (An approximate solution in non-resonance case is found in ref. [33]). Hence one usually mimic the exact expression by the linear "poor man" substitution:

$$i\dot{\rho} = ...(i/2)\left\{\Gamma,\, \rho - \rho_{eq}\right\}... \tag{7}$$

where curly brackets mean anti-commutator and the matrix $\Gamma$ effectively describes neutrino scattering and annihilation. It has the only non-vanishing entry at $aa$-corner in the flavor basis, which is taken as

$$\Gamma_{aa} \equiv \Gamma_0^a = C_\Gamma^a G_F^2 T^4 E \tag{8}$$

where $C_\Gamma^a$ is a constant. According to reference [34] $C_\Gamma^e = 1.27$ and $C_\Gamma^{(\mu,\tau)} = 0.92$, while according to [9], where slightly more accurate calculations were done, $C_\Gamma^e = 1.13$ and $C_\Gamma^{(\mu,\tau)} = 0.79$. This difference and even the absolute value of $\Gamma_{aa}$ are not essential for the magnitude of rising asymmetry because $\Gamma$ remains in some sense small and can be neglected (see below). Even the use of the approximate expression for the coherence breaking terms (7) in contrast to the exact collision integral which changes the results in the case of non-resonance oscillations by an order of magnitude [33], have a very weak impact on the value of the generated lepton asymmetry in the resonance case.

The matrix $\rho_{eq}$ is equal to $f_{eq}(\mu)\, I$ where $I$ is the unit matrix and and $f_{eq}(E/T, \mu)$ is the equilibrium Fermi distribution function:

$$f_{eq} = \frac{1}{\exp\left[(E-\mu)/T\right] + 1} \ . \tag{9}$$

where $\mu$ is neutrino chemical potential. In what follows we will take $\mu = 0$, though in refs. [4]-[8],[10] it was taken equal to the running value determined by the generated lepton asymmetry. However the final result for the asymmetry is not sensitive to this assumption (see discussion in section 5.

In the Friedman-Robertson-Walker universe the matrix elements of $\rho$ satisfy the following equations:

$$i(\partial_t - Hp\partial_p)\rho_{aa} = F_0(\rho_{sa} - \rho_{as})/2 - i\Gamma_0(\rho_{aa} - f_{eq}) \ , \tag{10}$$

$$i(\partial_t - Hp\partial_p)\rho_{ss} = -F_0(\rho_{sa} - \rho_{as})/2 \ , \tag{11}$$

$$i(\partial_t - Hp\partial_p)\rho_{as} = W_0\rho_{as} + F_0(\rho_{ss} - \rho_{aa})/2 - i\Gamma_0\rho_{as}/2 \ , \tag{12}$$

$$i(\partial_t - Hp\partial_p)\rho_{sa} = -W_0\rho_{sa} - F_0(\rho_{ss} - \rho_{aa})/2 - i\Gamma_0/2\rho_{sa} \ , \tag{13}$$

where $a$ and $s$ mean "active" and "sterile" respectively,

$$F_0 = \delta m^2 \sin 2\theta/2E, \tag{14}$$

$$W_0 = \delta m^2 \cos 2\theta/2E + V_{eff}^a, \tag{15}$$

$H = \sqrt{8\pi\rho_{tot}/3M_{Pl}^2}$ is the Hubble parameter, $M_{Pl}$ is the Planck mass, and $p$ is the neutrino momentum. The neutrinos are assumed to be very light, so that $p = E$. The total cosmological energy density is taken as $\rho_{tot} = 10.75\pi^2 T^4/30$. This corresponds to photons, $e^{\pm}$-pairs, and three types of neutrinos in thermal equilibrium with equal temperatures $T$. This approximation is quite good when $T$ is larger than a few hundred keV. At smaller temperatures $e^+e^-$-annihilation heats up electromagnetic component of the plasma and the temperatures of photons and neutrinos become different.

We have prescribed sub-index "0" to the coefficient functions in eqs. (10-13) to distinguish them from the same ones divided by $Hx$ (see eqs. (27-31) below).

The anti-neutrino density matrix, $\bar{\rho}$, satisfies the similar set of equations with the opposite sign of the antisymmetric term in $V^a_{eff}$ and with a slight difference in damping factors $\bar{\Gamma}_0$ which is proportional to the lepton asymmetry.

It is convenient to introduce new variables:

$$x = m_0 R(t) \text{ and } y = pR(t) \ , \tag{16}$$

where $R(t)$ is the cosmological scale factor so that $H = \dot{R}/R$ and $m_0$ is an arbitrary mass (just normalization), taken as $m_0 = 1$ MeV. One may approximately assume that $\dot{T} = -HT$, and correspondingly $R = 1/T$. In terms of these variables the differential operator $(\partial_t - Hp\partial_p)$ transforms to $Hx\partial_x$. We will normalize the density matrix elements to the equilibrium distribution with zero chemical potential:

$$\rho_{aa} = f_{eq}(y)\,[1 + a(x,y)],\ \ \rho_{ss} = f_{eq}(y)\,[1 + s(x,y)] \ , \tag{17}$$

$$\rho_{as} = \rho^*_{sa} = f_{eq}(y)\,[h(x,y) + il(x,y)] \ , \tag{18}$$

Other authors find it convenient to express the density matrix in terms of Pauli matrices and the polarization vector, $\vec{P} = (P_x, P_y, P_z)$, in such a way that:

$$\rho \equiv \frac{P_0}{2}\left[1 + \vec{P}\cdot\vec{\sigma}\right] \ , \tag{19}$$

The relation between different functions are $P_0 P_x = 2f_{eq}h$, $P_0 P_y = -2f_{eq}l$, $P_0 P_z = f_{eq}(a-s)$ and $P_0 = f_{eq}(2+a+s)$. In particular, $P_z = 1$ means that all the neutrinos are active, $\nu_e$, $\nu_\mu$, or $\nu_\tau$.

Let us now divide both sides of equations (10-13) by $Hx$ and denote the corresponding coefficient functions (14-15) with sub-index "1", e.g. $W_1 = W_0/Hx = U_1 \pm V_1 Z$, etc and thus we get:

$$U_1 = 1.12 \cdot 10^9 \cos 2\theta \delta m^2 \frac{x^2}{y} + \tilde{C}_2^a \frac{y}{x^4}, \tag{20}$$

$$V_1 = \frac{30}{x^2}, \tag{21}$$

$$Z = 10^{10} \left[ \frac{\eta_o}{12} + \int_0^\infty \frac{dy}{8\pi^2}\, y^2 f_{eq}(y)\, (a - \bar{a}) \right], \tag{22}$$

where $\delta m^2$ is expressed in eV$^2$ (here and below), $\tilde{C}_2^e \approx 26$ and $\tilde{C}_2^{\mu,\tau} \approx 7$ and $\eta_o$ is the charge asymmetry of all particles except for $\nu_a$ defined in accordance with eqs. (4,5). The normalization of the charge asymmetry term (22) is rather unusual and to understand the numerical values of the coefficients one should keep in mind the following. The coefficient $C_2$ in eq. (3) is found for the standard normalization of charge asymmetry with respect to the present day photon number density which differs from that in the early universe by the well known factor 11/4, related to the increase of photon number by $e^+e^-$-annihilation. On the other hand, lepton asymmetry, $L_{\nu_a}$, induced by neutrino oscillations, which is calculated in most of papers is normalized to the number density of photons that are in thermal equilibrium with neutrinos, so the factor 11/4 is absent. The photon number density is equal to $N_\gamma = 2\zeta(3) T_\gamma^3/\pi^2$ ($\zeta(3) \approx 1.2$). The charge asymmetry of neutrinos is

$$\eta_\nu = \frac{1}{4\zeta(3)} \left( \frac{T_\nu}{T_\gamma} \right)^3 \int dy y^2 (\rho_{aa} - \bar{\rho}_{aa}) \tag{23}$$

so that $\eta_{\nu_a} = 4L_{\nu_a}/11$. The quantity $Z$ introduced in eq. (22) differs from $L_{\nu_a}$ by the factor $2 \cdot 10^{-10} \pi^2/\zeta(3)$:

$$L = 16.45 \cdot 10^{-10}\, Z. \tag{24}$$

The factor $10^{-10}$ is chosen so that initially $Z = O(1)$. Noting that the charge asymmetry of neutrinos under study enters expressions (4,5) with coefficient 2 one obtains the coefficient $1/11.96 \approx 1/12$ in eq. (22).

Now, following ref. [9], we will introduce one more new variable $q = \xi_a x^3$ in such a way that $U_1$, eq. (20), vanishes at $q = y$ independently of the values of the oscillation parameters:

$$U_1 = 1.12 \cdot 10^9 \, \cos 2\theta \, |\delta m^2| \, \frac{x^2}{y} \left[ -1 + \left( \frac{y}{q} \right)^2 \right] \tag{25}$$

(it is assumed that $\delta m^2 < 0$). The coefficients $\xi_a$ are

$$\xi_e = 6.63 \cdot 10^3 \left( |\delta m^2| \cos 2\theta \right)^{1/2}, \quad \xi_{\mu,\tau} = 1.257 \cdot 10^4 \left( |\delta m^2| \cos 2\theta \right)^{1/2} \tag{26}$$

In what follows we assume that $\sin 2\theta \ll 1$ and $\cos 2\theta \approx 1$.

Written in terms of the variable $q$ the system of basic kinetic equations takes a very simple form [9]:

$$s' = -(K_a/y) \sin 2\theta \, l \tag{27}$$

$$a' = (K_a/y) \, (\sin 2\theta \, l - 2\gamma \, a) \tag{28}$$

$$h' = (K_a/y) \, (W l - \gamma \, h \,) \tag{29}$$

$$l' = (K_a/y) \, [\sin 2\theta \, (s - a)/2 - W h - \gamma \, l] \tag{30}$$

where $K_a = 1.12 \cdot 10^9 \, \cos 2\theta \, |\delta m^2|/3\xi_a$, so $K_e = 5.63 \cdot 10^4 (|\delta m^2| \, \cos 2\theta)^{1/2}$ and $K_{\mu,\tau} = 2.97 \cdot 10^4 (|\delta m^2| \, \cos 2\theta)^{1/2}$. In what follows we use the limit $K_a \gg 1$. This permits to make accurate analytic calculations. A large magnitude of this coefficient reflects a large frequency of neutrino oscillations with respect to other essential time scales. Its large value makes numerical solution very difficult but allows to make accurate analytical estimates.

The coefficient functions in these equations have the form:

$$W = U \pm y \, V \, Z, \quad U = y^2 \, q^{-2} - 1, \qquad V = b_a \, q^{-4/3}, \quad \gamma = \epsilon_a \, y^2 \, q^{-2} \tag{31}$$

where the signs $''-''$ or $''+''$ in $W$ refer to neutrinos and antineutrinos respectively; $b_e = 3.3 \cdot 10^{-3} (|\delta m^2| \cos 2\theta)^{-1/3}$, $b_{\mu,\tau} = 7.8 \cdot 10^{-3} (|\delta m^2| \cos 2\theta)^{-1/3}$, and $\epsilon_a$ are small coefficients, $\epsilon_e \approx 7.4 \cdot 10^{-3}$ and $\epsilon_{\mu,\tau} \approx 5.2 \cdot 10^{-3}$. Their exact numerical value is not important. It is noteworthy that the charge asymmetric term in $W$ comes with a very large coefficient if expressed in terms of $L$, $VZ \sim 10^7 q^{-4/3} L$, while in all other possible places $L$ (or chemical potential) enters with the coefficient of order 1.

It follows from eqs. (27-30) that

$$\partial_q \left(a^2 + s^2 + 2h^2 + 2l^2\right) = -4\gamma (K/y) \left(a^2 + h^2 + l^2\right) \tag{32}$$

so that the quantity in the r.h.s. may only decrease.

# 3 Solution of kinetic equations.

One can solve analytically the last two kinetic equations (29,30) with respect to $h$ and $l$ in terms of $a$ and $s$:

$$l(q,y) = -(K \sin 2\theta / 2y) \int_{q_{in}}^{q} dq_1 \left[a(q_1) - s(q_1)\right] e^{-\Delta\Gamma} \cos \Delta\Phi, \tag{33}$$
$$h(q,y) = -(K \sin 2\theta / 2y) \int_{q_{in}}^{q} dq_1 \left[a(q_1) - s(q_1)\right] e^{-\Delta\Gamma} \sin \Delta\Phi, \tag{34}$$

where $q_{in}$ is the initial "moment" $q$ from which the system started to evolve, $\Delta\Gamma = \Gamma(q,y) - \Gamma(q_1,y)$, $\Delta\Phi = \Phi(q,y) - \Phi(q_1,y)$, and

$$\partial_q \Gamma = K\gamma / y, \;\; \partial_q \Phi = KW/y. \tag{35}$$

We rewrite the first two equations (27,28) in terms of $\sigma = a + s$ and $\delta = a - s$:

$$\sigma' = -(K\,\gamma/y)\,(\sigma + \delta) \tag{36}$$
$$\delta' = (2K \sin 2\theta / y)\, l - (K\,\gamma/y)\,(\sigma+\delta) \tag{37}$$

The first of these equations can be solved for $\sigma$:

$$\sigma(q,y) = \sigma_{in}(y)\, e^{-\Gamma(q,y) + \Gamma_{in}(y)} - \frac{K}{y} \int_{q_{in}}^{q} dq_1 e^{-\Delta\Gamma} \gamma(q_1,y) \delta(q_1,y) \tag{38}$$

The first term in this expression, proportional, to the initial value $\sigma_{in}$ is exponentially quickly "forgotten" and we obtain the following equation that contains only $\delta$ (and another unknown function, integrated charge asymmetry $Z(q)$ that is hidden in the phase factor $\Delta\Phi$):

$$\delta'(q, y) = -\frac{K\,\gamma(q, y)}{y}\,\delta + \frac{K^2\,\gamma(q, y)}{y^2}\int_{q_{in}}^{q} dq_1 e^{-\Delta\Gamma}\gamma(q_1, y)\delta(q_1, y) \tag{39}$$

$$- \left(\frac{K\sin 2\theta}{y}\right)^2 \int_{q_{in}}^{q} dq_1 \delta(q_1, y) e^{-\Delta\Gamma}\cos\Delta\Phi \tag{40}$$

Up to this point this is an exact equation (with the omitted initial value of $\sigma$ which contribution is exponentially small). There is also an uncertainty related to the choice of the form of $\rho_{eq}$ in eq. (7) either with zero or non-zero chemical potential, see eq. (9). We have chosen here $\mu = 0$ and, as is argued in sec. 5, the choice $\mu \neq 0$ does not lead to a noticeably different results. This ambiguity could be rigorously resolved if one uses exact collision integrals instead of eq. (7). This will be discussed elsewhere.

Let us take now similar equation for antineutrinos and consider the sum and difference of these two equations for charge symmetric and antisymmetric combinations of the elements of density matrix, $\Sigma = \delta + \bar\delta$ and $\Delta = \delta - \bar\delta$. The equations have the following form:

$$\Delta' + \frac{K\gamma}{y}\Delta = \frac{K^2\gamma}{y^2}\int_{q_{in}}^{q} dq_1\, e^{-\Delta\Gamma}\gamma_1\,\Delta_1$$

$$- \left(\frac{K\sin 2\theta}{y}\right)^2 \int_{q_{in}}^{q} dq_1\, e^{-\Delta\Gamma}\left(\Sigma_1\frac{c-\bar c}{2} + \Delta_1\frac{c+\bar c}{2}\right) \tag{41}$$

$$\Sigma' + \frac{K\gamma}{y}\Sigma = \frac{K^2\gamma}{y^2}\int_{q_{in}}^{q} dq_1\, e^{-\Delta\Gamma}\gamma_1\,\Sigma_1$$

$$- \left(\frac{K\sin 2\theta}{y}\right)^2 \int_{q_{in}}^{q} dq_1\, e^{-\Delta\Gamma}\left(\Sigma_1\frac{c+\bar c}{2} + \Delta_1\frac{c-\bar c}{2}\right) \tag{42}$$

where sub-1 means that the function is taken at $q_1$, e.g. $\gamma_1 = \gamma(q_1, y)$, etc; $c = \cos\Delta\Phi$, and $\bar c = \cos\Delta\bar\Phi$. Using expressions (31, 35) we find:

$$\frac{c-\bar c}{2} = \sin\left[K\,(q-q_1)\left(-\frac{1}{y} + \frac{y}{q\,q_1}\right)\right]\,\sin\left[K\int_{q_1}^{q} dq_2 V(q_2)\,Z(q_2)\right] \tag{43}$$

$$\frac{c+\bar c}{2} = \cos\left[K\,(q-q_1)\left(-\frac{1}{y} + \frac{y}{q\,q_1}\right)\right]\,\cos\left[K\int_{q_1}^{q} dq_2 V(q_2)\,Z(q_2)\right] \tag{44}$$

Here $V(q)$ is given by expression (31) and does not depend on $y$.

At this stage we will do some approximations to solve the system (41,42). First, let us consider the terms proportional to $\gamma$. They are definitely not important at large $q$ (or low temperature). Let us estimate how essential are they at low $q$ ($q \sim 1$). Integrating by parts the first term in the r.h.s. of eq. (41), using expression (35) and neglecting exponentially small contribution of the initial value, we find:

$$\frac{K\gamma}{y}\Delta - \frac{K^2\gamma}{y^2}\int_{q_{in}}^{q} dq_1\, e^{-\Delta\Gamma}\gamma_1\,\Delta_1 = \frac{K\gamma}{y}\int_{q_{in}}^{q} dq_1 e^{-\Delta\Gamma}\,\frac{d\Delta_1}{dq_1} \tag{45}$$

The remaining integral can be easily evaluated in the limit of large $K\gamma/y = K\epsilon y/q^2$. Indeed, $\Delta\Gamma = K\epsilon y(q-q_1)/q\,q_1$ and for $q \leq 1$ the coefficient in front of the exponential $(q-q_1)$ is larger than 400 for $\nu_e$ and 300 for $\nu_\mu$ and $\nu_\tau$. So the integral strongly sits on the upper limit and, together with the coefficient in front, it gives just $\delta'(q)$. Thus, when the $\gamma$-terms are large they simply double the coefficient of $\Delta'$ in eq. (41): $\Delta' \to 2\Delta'$. A possible loophole in this argument is a very strong variation of the integrand, much stronger than that given by $\exp(\Delta\Gamma)$. However one can check from the solution found below that this is not the case.

Thus the role of $\gamma$ terms in eq. (41) is rather mild, they could only change the coefficient in front of $\Delta'$ from 1 to 2, and become negligible for large $q$ where the bulk of asymmetry is generated. So let us neglect these terms in the equation. This simplification does not have a strong impact on the solution.

Let us make one more approximate assumption, namely let us neglect the second term, proportional to $\Delta_1$, in the last integral of the r.h.s. of eq. (41). Initially $\Sigma = 2$ and $\Delta = 10^{-9} - 10^{-10}$ and the neglect of $\Delta$ in comparison with $\Sigma$ is a good approximation, at least at initial stage. We will check the validity of this assumption after we find the solution. And last, we assume that $\Sigma$ changes very slowly $\Sigma \approx \Sigma_{in} = 2$. Justification for the latter is a smallness of the mixing angle, $\sin 2\theta \sim 10^{-4}$. In the limit of zero mixing, the solution of eq. (42) is $\Sigma = const$. We will relax both these assumptions in sec. 4.

As a last step we need to find a relation between $\Delta = a - s - \bar{a} + \bar{s}$ and charge asymmetry $Z$. To this end one may use the conservation of the total leptonic charge:

$$\int_0^\infty dy\, y^2\, f_{eq}(y)\, (a + s - \bar{a} - \bar{s}) = const \tag{46}$$

Using this conservation law we find:

$$10^{10}\, \frac{d}{dq} \left[ \int_0^\infty dy\, y^2\, f_{eq}(y) \Delta(q, y) \right] = 16\pi^2 \frac{dZ}{dq} \tag{47}$$

Keeping all these assumptions in mind we can integrate both sides of eq. (41) with $dyy^2 f_{eq}(y)$ and obtain a closed ordinary differential equation for the asymmetry $Z(q)$, valid in the limit of large $K$. Integration over $y$ gives:

$$\begin{aligned} Z'(q) \quad = \quad & \frac{10^{10} K^2 (\sin 2\theta)^2}{8\pi^2} \int_0^\infty dy f_{eq}(y) \\ & \int_{q_{in}}^q dq_1 \exp\left[ -\frac{\epsilon y \zeta}{q q_1} \right] \sin\left[ \zeta \left( \frac{1}{y} - \frac{y}{q q_1} \right) \right] \sin\left[ bK \int_{q_1}^q dq_2 \frac{Z(q_2)}{q_2^{4/3}} \right] \end{aligned} \tag{48}$$

where $b$ is defined in eq. (31) and $\zeta = K(q - q_1)$. Integration over $y$ here can be done explicitly and the result is expressed through a real part of a sum of certain Bessel functions of complex arguments. To do that one has to expand

$$f_{eq} = \sum_n (-1)^{n+1} \exp(-ny) \tag{49}$$

and integrate each term analytically [35]. It can be seen from the result (it is more or less evident anyhow) that the integral over $q_1$ is saturated in the region $\zeta \sim 1$. So that we can take $q q_1 \approx q^2$ and

$$K \int_{q_1}^q dq_2 Z(q_2) q_2^{-4/3} \approx \zeta Z(q)/q^{4/3} \tag{50}$$

The correction in this expression is of the order of $Z'(q)/K$. It can be checked, using the solution obtained below, that the correction terms are indeed small.

Keeping this in mind we can take the integral over $\zeta$ in the r.h.s. of eq. (48) . To ensure convergence we proceed as follows. First, we expand $f_{eq}(y)$ in accordance with eq. (49). Each term of the series is non-singular in the complex $y$-plane. After that we can rotate the contour in the complex $y$-plane to imaginary axis, clockwise for the

$\exp[i\zeta(1/y - y/q^2)]$-part of $\sin[\zeta(1/y - y/q^2)]$, $(y \to -iy)$, and counter-clockwise for the complex conjugate part $(y \to iy)$. Both terms give $\exp[-\zeta(1/y + y/q^2)]$, so the integral over $\zeta$ is exponentially converging for each term in the series (49) and the resulting series is convergent as well:

$$
\begin{aligned}
Z'(q) \;=\;& \frac{10^{10} K (\sin 2\theta)^2}{8\pi^2} \sum_n (-1)^{n+1} \int_0^\infty \frac{dy}{4i} \\
& \left[ e^{iny} \left( \frac{1}{\frac{1}{y} + \frac{y(1-i\epsilon)}{q^2} - i\phi} - \frac{1}{\frac{1}{y} + \frac{y(1-i\epsilon)}{q^2} + i\phi} \right) \right. \\
& \left. + e^{-iny} \left( \frac{1}{\frac{1}{y} + \frac{y(1+i\epsilon)}{q^2} - i\phi} - \frac{1}{\frac{1}{y} + \frac{y(1+i\epsilon)}{q^2} + i\phi} \right) \right]
\end{aligned}
\tag{51}
$$

where $\phi \equiv VZ = bZ(q)q^{-4/3}$.

Since $\epsilon \sim 10^{-2}$ is a small number it may be neglected and changing the integration variable, $y = qt$, we come to the expression:

$$
Z'(q) = \frac{10^{10} K (\sin 2\theta)^2}{8\pi^2} q^2 \chi(q) \sum_n (-1)^{n+1} \int_0^\infty \frac{dt\, t^2 \cos(nqt)}{(1+t^2)^2 + t^2\, \chi^2(q)} \tag{52}
$$

where $\chi(q) = q\phi(q) = bZ(q)q^{-1/3}$. Both the integral over $t$ and summation over $n$ can be done explicitly and we finally obtain:

$$
\kappa\, Z' = \frac{10^{10} K (\sin 2\theta)^2}{16\pi} \frac{q^2}{\sqrt{\chi^2 + 4}} \left[ t_2 f_{eq}(qt_2) - t_1 f_{eq}(qt_1) \right] \tag{53}
$$

where we introduced the coefficient $\kappa$, such that $\kappa = 1$ for large $q$ and $\kappa = 2$ for $q \sim 1$. It reflects the role of decoherence terms, proportional to $\gamma$, see discussion after eq. (45). The quantities $t_{1,2}$ are the poles of the denominator in eq. (52) in the complex upper half-plane of $t$ (resonances):

$$
t_{1,2} = \frac{\sqrt{\chi^2 + 4} \pm \chi}{2} \tag{54}
$$

It is an ordinary differential equation that can be easily integrated numerically. It quite accurately describes evolution of the lepton asymmetry in the limit when back reaction may be neglected: we assumed above that $\Sigma = 2$ and $\Delta \ll \Sigma$.

Before doing numerical integration let us consider two limiting cases of $q$ close to initial value when asymmetry is very small and the case of large $q$. When $q$ is not too large, $q \sim 1$, the r.h.s. of eq. (53) can be expanded in powers of $\chi$ and we obtain a very simple differential equation that can be integrated analytically:

$$Z' = \frac{10^{10} K (\sin 2\theta)^2}{64\pi} q^2 f_{eq}(q) \chi(q) \left[-1 + q\left(1 - f_{eq}(q)\right)\right] \tag{55}$$

(we took here $\kappa = 2$). One sees that for $q < q_{min} = 1.278$ the asymmetry exponentially decreases and reaches the minimum value

$$\frac{Z_{min}}{Z_{in}} = \exp\left[-\frac{10^{10} K (\sin 2\theta)^2 b}{64\pi} \int_0^{q_{min}} dq\, q^{5/3} f_{eq}(q) \left(1 - \frac{q}{1 + \exp(-q)}\right)\right]. \tag{56}$$

The integral in the expression above is equal to 0.07539 and e.g. for $(\nu_e - \nu_s)$-oscillations the initial asymmetry drops by 3 orders of magnitude in the minimum. The drop would be significantly stronger even with a mild increase of mixing angle or mass difference. The temperature, when the minimum is reached (corresponding to $q_{min} = 1.278$) is

$$T^{e}_{min} = 17.3 \left(\delta m^2\right)^{1/6}\ \mathrm{MeV},\ \ T^{\mu,\tau}_{min} = 23.25 \left(\delta m^2\right)^{1/6}\ \mathrm{MeV} \tag{57}$$

These results rather well agree with ref. [13] for $\nu_e$, while agreement for $\nu_\mu$ and $\nu_\tau$ case (see e.g. the papers [6, 32, 10]) is not so good.

For $q > q_{min}$ the asymmetry started to rise exponentially and this regime lasted till $\chi$ becomes larger than one and the asymmetry reaches the magnitude $Z \sim 10^3$ or $L \sim 10^{-6}$. After that the asymmetry started to rise as a power of $q$. For large $q$ and $\chi$ the term containing $t_2$ dominates the r.h.s. of eq (53) and now it takes the form:

$$Z^2\, Z' = \frac{10^{10} K (\sin 2\theta)^2}{16\pi\, b^2} q^{8/3} f_{eq}(1/VZ) \tag{58}$$

where $V$ is given by eq. (31). Assuming that $VZ$ is a slowly varying function of $q$ we can integrate this equation and obtain:

$$Z(q) \approx 1.6 \cdot 10^3 q^{11/9}\ \text{ or }\ L(q) \approx 2.5 \cdot 10^{-6} q^{11/9} \tag{59}$$

The concrete values of numerical coefficients above are taken for $(\nu_e - \nu_s)$-oscillations with $\delta m^2 = -1\mathrm{eV}^2$ and $\sin 2\theta = 10^{-4}$. This result is in a good agreement with the numerical solution of eq. (53) and the functional dependence, $L \sim q^{11/9} \sim T^{-11/3}$, agrees with that found in ref. [11] and slightly disagrees with the results of refs. [4]-[6],[10, 13] where the law $L \sim q^{4/3} \sim T^{-4}$ was advocated.

Numerical solution of eq. (53) is straightforward. It well agrees with the simple analysis presented above. In the power law regime, where the bulk of asymmetry is generated, it is accurately approximated by the found above law (59):

$$L_e = 2.5 \cdot 10^{-6} C_e \, q^{11/9} \tag{60}$$

For $\nu_e - \nu_s$ mixing with $(\sin 2\theta)^2 = 10^{-8}$ and $\delta m^2 = -1$ the correction coefficient $C_e$ is 0.96, 0.98, 1, 1.01, and 0.997 for $q = 6630,\ 1000,\ 100,\ 10$, and 5 respectively. The results of numerical solution well agree with those of ref. [13] in the temperature range from 10 down to 1 MeV for $\nu_e - \nu_s$ case.

For the $(\nu_\mu - \nu_s)$-mixing with $\delta m^2 = -10$ and $(\sin 2\theta)^2 = 10^{-9}$ the solution can be approximated as

$$L_\mu \approx 1.2 \cdot 10^{-6} C_\mu \, q^{11/9} \tag{61}$$

with the correction coefficient $C_\mu = 0.84$, 0.9, 0.98, 1.02, 1.05, 1 for $q = 4 \cdot 10^4$, $10^4$, $10^3$, $10^2$, 10, 5 respectively. These results reasonably well agree with the calculations of ref. [6] in the temperature range from 25 down to 2 MeV. At smaller temperatures this power law generation of asymmetry must stop and it abruptly does, according to the results of the quoted papers, but the solutions of eq. (53) continue rising because back reaction effects are neglected there. We will consider these effects in the next section.

# 4 Back reaction.

The solution obtained above should be close to the exact one if $\Delta(q, y) \ll \Sigma(q, y)$ and $\Sigma \approx 2 = const$, see eq. (41). Since now we know the function $Z(q)$ we can find

$\Delta(q,y)$ and check when this assumptions are correct. We will consider the region of sufficiently large $q$ when the second pole (resonance) $t_1$ in eq. (53) is not important. Its contribution is suppressed as $\exp(-q^2VZ)$ and it may be neglected already at $q>5$ (we assume for definiteness that initial value of the asymmetry is positive, otherwise the role of the two poles would interchange). In terms of oscillating coefficients $\cos\Delta\Phi$ or $\cos\bar{\Delta}\Phi$, entering eq. (41), it means that only one of them is essential. It has a saddle point where the oscillations are not too fast, while the other quickly oscillates in all essential region of momenta $y$. With our choice of the sign of the initial asymmetry only $\cos\bar{\Delta}\Phi$ has an essential saddle point and in this approximation the equation (41) can be written as:

$$\Delta'(q,y) = -\frac{K^2(\sin 2\theta)^2}{y^2}\int_0^q dq_1 \cos\bar{\Delta}\Phi \tag{62}$$

For large $q$ the phase difference is equal to

$$\Delta\Phi = K\left(-\frac{q-q_1}{y} + \int_{q_1}^q dq_2 V(q_2)Z(q_2)\right) \tag{63}$$

This integral can be taken in saddle point approximation. To this end let us expand:

$$\Phi(q,y) = \Phi(q_R,y) + \frac{(q-q_R)^2}{2}\Phi''(q_R,y) \tag{64}$$

where the saddle (resonance) point $q_R$ is determined by the condition

$$\Phi'(q_R,y) = K\left(VZ - \frac{1}{y}\right) = 0 \tag{65}$$

For $q<q_R$ the integral in the r.h.s. of eq. (62) may be neglected, while for $q>q_R$ it is

$$\int_0^q dq_1 \cos\bar{\Delta}\Phi \approx \theta(q-q_R)\,\mathcal{R}e\left\{\sqrt{\frac{2\pi}{|\Phi''(q_R,y)|}}\; e^{[\Phi(q,y)-\Phi(q_R,y)-i\pi/4]}\right\} \tag{66}$$

where $\Phi'' = (VZ)'$.

Now repeating similar integration in eq. (62) we can easily find $\Delta(q,y)$:

$$\Delta(q,y) = \theta(q-q_R)\frac{\pi\, K(\sin 2\theta)^2}{y^2|(VZ)'|_R} \tag{67}$$

Note the factor 1/2 that comes from the theta-function in expression (66). It permits integration only over positive values of $(q - q_R)$.

From the saddle point condition follows

$$(VZ)' = VZ \left( \frac{V'}{V} + \frac{Z'}{Z} \right)_R = \frac{1}{y} \left( \frac{V'}{V} + \frac{Z'}{Z} \right)_R = -\frac{1}{9yq_R} \tag{68}$$

In the last equality the solution $Z \approx 1.5 \cdot 10^3\, q^{11/9}$ and $V = bq^{-4/3}$ were used. From the condition $(VZ)' = 1/y$ we find

$$q_R \approx (5y)^9 \tag{69}$$

where we used $\delta m^2 = -1$ and $b_e = 3.31 \cdot 10^{-3}$.

As a simple check we may calculate the integrated lepton asymmetry $L(q) = 16.45 \cdot 10^{-10} Z(q)$:

$$L(q) = \frac{16.45}{16\pi^2} \int_0^\infty dy y^2 f_{eq}(y) \Delta(q,y) = \frac{148K(\sin 2\theta)^2}{16\pi} \int_0^{y_{max}} dy\, y\, (5y)^9 f_{eq}(y) \tag{70}$$

where $y_{max} = q^{1/9}/5$. This integral can be easily calculated and the result is in a good agreement with eq. (59), as one should expect.

However the magnitude of $\Delta(q,y)$ is too large. For example at $q = 6630$ (corresponding to $T = 1$ MeV for $(\nu_e - \nu_s)$-oscillations) we find $\Delta = 9\pi K(\sin 2\theta)^2 q_R/y \approx 200 \gg 1$. It violates the condition (32) and contradicts the assumption that $\Delta \ll \Sigma$. Naively one would expect that the asymmetry should be suppressed by two orders of magnitude but as we will see below it is not the case. The evolution of $Z(q)$ changes, due to the back reaction, and the behavior of the resonance $y_R(q)$ also becomes different from $y_R = q^{1/9}/5$ found above.

If only one resonance is essential then $(\Sigma + \Delta)$ is conserved, as is seen from eqs. (41,42) with $\cos \Delta\Phi \to 0$. It corresponds to conservation of the total leptonic charge if oscillations are efficient only in neutrino (or antineutrino) channel. In this case we come to the equation:

$$\Delta'(q,y) = -\frac{K^2(\sin 2\theta)^2}{y^2} \int_0^q dq_1 \cos \bar{\Delta}\Phi \, [1 - \Delta(q_1, y)] \tag{71}$$

This integral can be taken in the same way as above in the saddle point approximation and we obtain:

$$\Delta(q,y) = \theta(q - q_R)\lambda\left[1 - \Delta(q_R, y)\right] \tag{72}$$

where the last term describes the back-reaction and

$$\lambda = \frac{\pi K (\sin 2\theta)^2}{y^2 |(VZ)'|_R} \tag{73}$$

The derivative of $VZ$ is taken over $q$ at $q = q_R(y)$ found from the resonance condition $V(q_R)Z(q_R) = 1/y$.

Since $\theta(0) = 1/2$, we find

$$\Delta(q,y) = \frac{2\lambda}{2+\lambda}\,\theta(q - q_R) \tag{74}$$

With this expression for $\Delta(q,y)$ we can find integrated asymmetry

$$Z(q) = \frac{10^{10}}{16\pi^2}\int_0^{y_R} dy y^2 f_{eq}(y)\,\frac{2\lambda}{2+\lambda} \tag{75}$$

where $y_R(q) = 1/[V(q)Z(q)]$. This equation can be reduced to an ordinary differential equation in the following way. Let us introduce the new variable

$$\tau = \frac{1}{VZ} \tag{76}$$

and consider the new unknown function $q = q(\tau)$. Correspondingly $Z = q^{4/3}(\tau)/(b\tau)$. The derivative over $q$ should be rewritten as:

$$\frac{d(VZ)}{dq} = -\frac{1}{\tau^2}\left(\frac{dq}{d\tau}\right)^{-1} \tag{77}$$

Under the sign of the integral over $y$ one should take $\tau = y$, while the upper integration limit is $y_{max} = \tau$. Now we can take derivatives over $\tau$ of both sides of the equation (75) and obtain:

$$\frac{d}{d\tau}\left[\frac{q^{4/3}(\tau)}{b\tau}\right] = \frac{10^{10}}{16\pi^2}\,\frac{2\lambda}{2+\lambda}\,\tau^2 f_{eq}(\tau). \tag{78}$$

where now $\lambda = \pi K (\sin 2\theta)^2\, dq/d\tau$. This is the final equation for determination of the integrated asymmetry with the account of the back reaction. As initial condition we take the magnitude of asymmetry found from solution of eq. (53) at $q = 5$. At this $q$ the back reaction is still small but already the regime of one pole dominance begins. Under the latter assumption the above equation (78) is derived.

The numerical solution of eq. (78) is straightforward. In particular, if one neglects $\lambda$ with respect to 2 in the denominator of the r.h.s. of this equation, then its numerical solution gives exactly the same result for the asymmetry as has been found above from eq. (53). An account of back reaction is not essential at high temperatures but it is quite important in the temperature region of Big Bang Nucleosynthesis. In particular, at $T = 1$ MeV the asymmetry is $L = 0.0435$ and at $T = 0.5$ MeV it is $L = 0.25$. These values are approximately 3 times smaller than those found without back reaction. Asymptotic constant value of $L$ is reached at $T < 0.3$ MeV and is equal to 0.35. These numerical values are found for electronic neutrinos with $\delta m^2 = -1$ and $(\sin 2\theta)^2 = 10^{-8}$. The asymptotic value is approximately twice smaller than that presented in the paper [13], the same is true for the magnitude of the asymmetry in the nucleosynthesis region as well. Another important effect for BBN is the shape of the spectrum of electronic neutrinos that may noticeably deviate from the simple equilibrium one given by expression (9) even with a non-zero chemical potential. It will be considered elsewhere.

For $\nu_\mu$ or $\nu_\tau$ with $\delta m^2 = -10$ and $(\sin 2\theta)^2 = 10^{-9}$ the asymmetry $L_\mu$ asymptotically tends to 0.237 in a good agreement with ref. [6]. For non-asymptotic values of temperature the corrected by back reaction asymmetry is $L_\mu = 6.94 \cdot 10^{-3}$ for $T = 3$ MeV (1.24 smaller than non-corrected one), $L_\mu = 0.025$ for $T = 2$ MeV (1.48 smaller), and $L_\mu = 0.164$ for $T = 1$ MeV (2.6 times smaller).

There is an easy way to find the asymptotic constant value of the asymmetry, $Z_0$ or $L_0$. To this end is convenient to use eq. (75) with a constant $Z$:

$$\lambda = \frac{3\pi K (\sin 2\theta)^2 b^{3/4} Z_0^{3/4}}{4 y^{1/4}} = 70.85 (\delta m^2)^{1/4} (\sin 2\theta / 10^{-4})^2 L_0^{3/4} \tag{79}$$

This result is the same both for $\nu_e$ and $\nu_{\mu,\tau}$.

Since the upper limit of the integral over $y$ is $y_{max} = q^{4/3}/(bZ_0) \gg 1$ for large $q$, we obtain the following equation for $L_0$:

$$L_0 = 0.208 \int_0^\infty dy y^2 f_{eq}(y) \frac{\lambda}{2+\lambda} \tag{80}$$

Numerical solution of this equation gives $L = 0.35$ for $\delta m^2 = -1$ and $(\sin 2\theta)^2 = 10^{-8}$ and $L = 0.27$ for $\delta m^2 = -10$ and $(\sin 2\theta)^2 = 10^{-9}$ in a good agreement with the solution of differential equation (78).

# 5 Discussion.

Thus, the statement of a generation of a large lepton asymmetry by oscillations between active and sterile neutrinos is essentially confirmed here. The agreement between the present calculations and those of ref. [6] for the case of $\nu_\mu$ and $\nu_s$ mixing is very good, while there is a noticeable difference between the results of the present paper and the paper [13] for the case of $\nu_e$ and $\nu_s$ mixing. In the temperature range important for the primordial nucleosynthesis the results of the present paper is 2-3 times smaller.

There are some other minor differences. According to our results the asymmetry rises as $T^{11/3}$, while the fit to numerical solution of the papers quoted in the text is $T^4$. This difference is very important for the evolution of the resonance value of the neutrino momentum $y$ and may possibly explain the difference of the results. On the other hand, if this is the case, a good agreement for the $\nu_\mu - \nu_s$ case becomes mysterious.

Another difference between the present approach and the calculations of refs. [4]-[8],[10, 13] is the treatment of repopulation of active neutrino states. In those papers the equilibrium number density of active neutrinos was taken in the form (9) with a non-zero chemical potential which value is found in a self-consistent way together with calculation of the lepton asymmetry. This may be true when the decoherence effects, described by $\gamma$, are strong. But the effects of neutrino production were important only at high temperatures, when a minor fraction of asymmetry was generated. At that stage the contribution from non-zero $\mu$ into kinetic equations are negligible in comparison with the terms coming from $Z$ in effective potential, the latter are amplified by the factor $10^7$. Moreover, if the effects of $\mu$ were non-negligible, one should take into account similar effects that lead to the difference between $\gamma$ and $\bar{\gamma}$ of the same magnitude. The agreement between the calculations for the $(\nu_\mu - \nu_s)$ case discussed above shows that one may neglect $\mu$ in the equilibrium distribution function.

The calculations presented here does not show any chaoticity. To be more precise numerical solution of equation (53) shows chaotic behavior with an increasing $K \sin 2\theta$. However this chaoticity is related to numerical instability because with the increasing coefficient in front of the r.h.s. of the equation the minimal value of the asymmetry becomes very small and can be smaller than the accuracy of computation. In this case the numerically calculated value of the asymmetry may chaotically change sign. However this regime is well described analytically and it can be seen that the sign of asymmetry does not change. There could be another effect first discussed in ref. [18] (see also [21]) - small primordial fluctuations of the cosmological (baryonic) charge asymmetry could be amplified by oscillations. This effect is related to neutrino diffusion and has nothing to do with the discussed chaoticity. A chaotic behavior was observed in several recent papers [16, 17, 20] in a simplified approach when the kinetic equations were solved for a fixed "average" value of neutrino momentum, $y = 3.15$, so that integro-differential equations become much simpler ordinary differential ones. However many essential features of the process could be obscured in this approach, in particular "running of the resonance" over neutrino spectrum and it is difficult to judge how reliable are the results. Moreover, the average value of $1/y$ that enters the refraction index, is $\langle 1/y \rangle \approx 1$ and not $1/\langle y \rangle \approx 0.3$, though this numerical difference might not be important for the conclusion.

Possibly fixed-momentum approach is not adequate to the problem as can be seen from a very simple example. Let us consider oscillations between $\nu_a$ and $\nu_s$ in vacuum. Then for a neutrino with a fixed energy the leptonic charge in active neutrino sector would oscillate with a very large frequency, $\sim \sin(\delta m^2 t/E)$. However if one averages this result with thermal neutrino spectrum, the oscillations of leptonic charge would be exponentially suppressed. Still this counter-example is also oversimplified and cannot be considered as a rigorous counter-argument. It may happen for example that in the case of smaller $K$ when saddle point does not give a good approximation, the differential asymmetry $\Delta(q, y)$ may be an oscillating function but the integrated (total) asymmetry is smooth. Another logically open option is that for a smaller $K$ the asymmetry might be chaotic but not large. Momentum dependent equations were analyzed in ref. [19] and a chaoticity for some values of parameters were observed but the authors could not exclude its origin by an instability of numerical computation procedure. At this stage is difficult to make a final judgment.

As we have already mentioned the semi-analytic solution found in this paper confirms a strong generation of lepton asymmetry. The results obtained are accurate in the limit of large values of parameter $K$ and for sufficiently small mixing, $(\sin 2\theta)^2 < 0.01 - 0.001$. For a larger mixing the population of sterile states becomes very strong and the $\nu_s$ and $\bar{\nu}_s$ states closely approach equilibrium and become equally populated, so the asymmetry is not generated. For small values of the product $K \sin 2\theta$ the process of asymmetry generation is not efficient and the net result is rather low. Numerical calculations of the effect for very low values of the mass difference $\delta m^2 = 10^{-7} - 10^{-11}$ eV$^2$ show that the asymmetry could rise only up to 4 orders of magnitude [36]-[38] producing the net result at the level of $10^{-5}$. According to the calculations of this work, the asymmetry strongly rises if $\delta m^2 > 10^{-3}$ eV$^2$ and possibly for smaller values depending upon the mixing angle. More accurate estimates of the parameter range where the asymmetry may strongly rise and the role of the asymmetry generation in big bang nucleosynthesis will be discussed elsewhere.

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